Introduction to particle-in-cell methods for the simulation of the Vlasov-Maxwell gyrokinetic equations

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The particle-in-cell (PIC) algorithm is the most popular method for the simulation of the general 6-D Vlasov-Maxwell problem and it is widely used also for the simulation of the 5D gyrokinetic equations. The method consists of coupling a particle based algorithm for the Vlasov equation with a grid based method for the computation of the self-consistent electromagnetic fields. The distribution function is discretised by a collection of macro-particles, or markers, representing the initial distribution function. The markers are then advanced in time by solving the characteristic equations (gyrokinetic Euler-Lagrange equations). The self-consistent electromagnetic fields, needed to advance in time the marker positions and velocities, are computed by calculating the sources of Maxwell's equations (charge densities and currents) directly from the markers by projecting them on a 3D grid in real space. Finite elements, finite volumes, finite difference or any other classical method can be used to solve the gyrokinetic Poisson and Ampère equations on the grid. The electromagnetic fields then need to be computed at the marker positions, using some interpolation procedure and can be used to further advance the markers in time, thus closing the time loop.

The PIC method can be interpreted as a probabilistic Monte-Carlo like method, consisting of calculating integrals of the continuous distribution function using a finite set of discrete markers. Even when the markers are initialized in a non random manner, the nonlinear interactions along with numerical errors introduce random effects after some time. Therefore, the same tools for error analysis and error reduction used in Monte-Carlo numerical methods can be applied to PIC simulations.

The lecture will cover the basics of the PIC method for the discretisation of the electromagnetic gyrokinetic Vlasov-Maxwell equations, focusing in particular on statistical error analysis and error reduction techniques.