Monte Carlo implementation of a guiding-center Fokker-Planck kinetic equation

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The kinetic equation is first introduced simply as the continuity equation for the probability density in the 6-dimensional phase space. In the presence of only external, macroscopic fields this is called the Vlasov equation.

In a plasma, the charged particles are also subjected to the microscopic fluctuations in the electromagnetic fields produced by other particles, leading to stochastic motion. Therefore a probabilistic view on the statistical description of the plasma can be adopted. Typically, in fusion plasmas, small-angle collisions dominate, and the evolution of the distribution function can be approximated by an equation of the Fokker-Planck form in 6 dimensions.

Solving the 6-dimensional partial differential equation with FEM or finite difference methods is highly inconvenient, to say the least, but luckily there is an alternative: The problem can be solved by integrating corresponding stochastic differential equation using Monte Carlo method and constructing the solution as an average of the simulated stochastic paths. One of the major advantages of the Monte Carlo method is that it allows for arbitrary shape for the simulation domain.

In problems related to magnetic confinement, integrating the full particle motion is computationally very expensive due to the rapid gyro motion that requires small time steps. In most applications, the information of the gyro phase is redundant, and it is sufficient to follow just the particle *guiding center*. The fast gyro motion around the field lines can be eliminated using appropriate Lie transformations that isolate the time evolution of the gyro phase. This is called the guiding-center transformation, and it is applied to the kinetic Fokker-Planck equation. In this context, we introduce the Poisson bracket formalism that facilitates consistent transformation of the kinetic equation. With the gyro motion time scale eliminated, the stochastic differential equation for the guiding center formalism is given.

The presentation is finished with a few applications of the outlined formalism, also indicating situations where the guiding-center formalism becomes insufficient.