Integrated Modelling and Simulation of Toroidal Plasmas

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1. Integrated modelling of toroidal plasmas
2. Data exchange in integrated simulation
3. Integrated tokamak modelling code TASK
4. Various level of transport modelling
5. Full wave analysis in toroidal plasmas
6. Summary
Integrated Simulation of Toroidal Plasmas

In order to

- predict the performance of future fusion devices
- optimize their operation scenario
- contribute to acceptable design of DEMO reactor

We need a reliable tool to describe

**Whole plasma**
- core, edge, scrape-off layer, divertor plasmas, and plasma-wall interactions

**Whole discharge period**
- startup, sustainment, probabilistic incidents, and shut down
## Use Case of Integrated Modelling

<table>
<thead>
<tr>
<th>Phase</th>
<th>Activities</th>
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</table>
| Device design phase          | Prediction of performance  
                               | Specification of components                                                 |
| Before experiment            | Prediction of time evolution  
                               | Optimization of operation scenario                                          |
| During experiment            | Real time analysis  
                               | Between shot analysis                                                      |
| After experiment             | Systematic analysis of experimental data  
                               | Validation of physics models                                                |
| Next device                  | Conceptual design  
                               | Development of control system                                               |
Modelling of Toroidal Plasmas

- Energetic Ion Confinement
- Transport Barrier
- Turbulent Transport
- Impurity Transport
- SOL Transport
- Divertor Plasma
- Core Transport
- Non-Inductive CD
- RF Heating
- NBI Heating
- Neutral Transport
- Plasma-Wall Int.
- Blanket, Neutronics, Tritium, Material, Heat and Fluid Flow

**One simulation code cannot cover all range.**

**Wide range of time scale, spatial scale, and understanding**

- Integrated simulation combining modelling codes
- Various levels of physics model
Structure of Integrated Modelling

Integrated Modeling

- Equilibrium:
  - Rotation, Anisotropy
  - Magnetic Island

- Core Plasma Transport:
  - Neoclassical Transport
  - Neutral Transport
  - Turbulent Transport
  - Impurity Transport
  - Radiation Transport
  - Energetic Particles

- Global Stability:
  - Linear Stability
  - Plasma Deformation

- Source:
  - Wave: IC, LH, EC
  - Pellet
  - NBI

- Peripheral Plasma Transport:
  - Collisionsal Transport
  - Neutral Transport
  - Turbulent Transport
  - Impurity Transport
  - Plasma-Wall Interaction
  - Radiation Transport

- Control model

- Diagnostic model

3D MHD Equilibrium

Turbulence Simulation

Nonlinear MHD

Material Simulation

Equilibrium:

- Fixed boundary
- Free boundary
- Time evolution
- Diffusive transport
- Dynamic transport
- Kinetic transport
- Particle orbit

Global Stability:

- MHD stability
- Kinetic stability
- Ray/beam tracing
- Full wave
- Pellet ablation

Source:

- Fluid model
- Kinetic model
- Particle model
Desired features of Integrated Code

★ Modular structure
  † Easier maintenance of components
    • Addition of new models, update of old models
  † Various levels of analyses:
    • Quick, Standard, Precise, Rigorous

★ Unified interface
  † Data set for information exchange
  † Program interface for data exchange
  † File interface for data storage
  † User interface for easier learning

★ High usability
  † Portability: Various computational environment
  † Source accessibility: More user, easier maintenance
  † Visualization: Understanding of phenomena

★ High performance
  † Parallel processing for large-scale and fast computation
Integrated Modelling Activities

★ JA: BPSI
  ‣ Burning Plasma Simulation Initiative
  ‣ Data structure and data interface: BPSD
  ‣ Execution control interface: BPSX

★ EU: ITM TF
  ‣ Integrated Tokamak Modelling - Task Force
  ‣ Data model: CPO (Consistent Physical Objects)
  ‣ Code interface: UAL (Universal Access Layer)

★ ITER: IM Programme
  ‣ IMAS: Integrated Modelling Analysis Suits
  ‣ IM standards and guideline
  ‣ ITER Data model
    • Data exchange between modules
    • Description of device (coils, actuators, diagnostics)
    • Experimental and simulation data storage
Data exchange between components: BPSD

* Purpose
  - **Standard dataset:** Specify set of data
  - **Specification of data exchange interface:** initialize, set, get
  - **Specification of file i/o interface:** save, load

* Policy of BPSD
  - **Minimum and Sufficient Dataset**
    - To minimize the data to be exchanged
    - Mainly profile data
    - Routines to calculate global quantities
  - **Minimum Arguments in Interfaces**
    - To maximize flexibility
    - Use structured data
    - Only one dataset in the arguments of an interface
  - **Minimum Kinds of Interfaces**
    - To make modular programming easier
    - Use function overloading
BPSD Data Exchange Interface

- **Standard dataset**: Specify data to be stored and exchanged
  - **Data structure**: Derived type (Fortran95): structured type
    - time
    - number of grid
    - number of species
    - normalized radius
    - Species specifier
    - plasma density
    - plasma temperature
    - e.g.  
      - plasmaf\%time
      - plasmaf\%nrmax
      - plasmaf\%nsamax
      - plasmaf\%rho(nr)
      - plasmaf\%ns(nsa)
      - plasmaf\%data(nr,nsa)\%density
      - plasmaf\%data(nr,nsa)\%temperature

- **Specification of API**:
  - **Program interface**
    - **Set data**
    - **Get data**
    - **Save data to file**
    - **Load data from file**
      - e.g.  
        - bpsd_set_data(plasmaf,ierr)
        - bpsd_get_data(plasmaf,ierr)
        - bpsd_save(ierr)
        - bpsd_load(ierr)
  - **BPSD data file** (bpsddata): Binary file of all existing bpsd data
### BPSD Standard Dataset

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<th>Category</th>
<th>Name</th>
<th>EQ</th>
<th>TR</th>
<th>TX</th>
<th>FP</th>
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BPSD Code Interface

* bpsd_set_data(data,ierr):
  ‣ Copy data into internal dataset

* bpsd_get_data(data,ierr):
  ‣ Copy of interpolate data from internal dataset
    • If nrmax=0, copy data;
    • otherwise interpolate for given mesh.

* bpsd_save(ierr):
  ‣ Save all BPSD data into a file
    ‣ Name of the file is optional.

* bpsd_load(ierr):
  ‣ Load all BPSD data from a file
    ‣ Name of the file is optional.

* Interface for history archiving is under consideration.
**Several Approaches on Workflow**

- **Monolithic code approach:** original approach
  - Memory-based data exchange
    - Template: `call bpsd_get_data`
    - Calculation
    - `call bpsd_set_data`

- **Command approach:** for script and workflow tool
  - File-based data exchange
    - Template: `call bpsd_load ← bpsddata`
    - `call bpsd_get_data`
    - Calculation
    - `call bpsd_set_data`
    - `call bpsd_save → bpsddata`

- **Pre- and post-process approach:** no modification of the code
  - Data conversion
    - Template
      - `pre-process: bpsddata → input file`
      - `run code`
      - `post-process: output file → bpsddata`
Integrated Modelling Code: TASK

Transport Analyzing System for tokamak

- Core of Integrated Modelling Code in BPSI
  - Modular structure for easier maintenance
  - Reference implementation of BPSD and BPSX
- Various Heating and Current Drive Scheme
  - EC, LH, IC, AW, NB
- High Portability
  - Most of library routines included
  - Original graphic libraries (X11, Postscript, OpenGL, SVG)
- Development using CVS (Version control for collaboration)
- Open Source: http://bpsi.nucleng.kyoto-u.ac.jp/task/
- Parallel Processing using MPI and PETSc
Present Structure of the TASK code and related codes

Developed since 1992, now at Kyoto University
Present Structure of TASK3D for Helical Plasmas

Data Interface

Profile Database

International Tokamak Profile DB

JT-60U Exp. Data

LHD Exp. Data

Simulation DB

BPSD

PL

EQ

EX

TR

TX

T2

FP

WR

WM

WF

DP

FIT3D

EG

EI

ER

DCOM/NNW

Task

2D fixed-/free-boundary equilibrium

2D anisotropic pressure equilibrium

1D diffusive transport

1D dynamiccal transport

2D dynamiccal transport

1D kinetic transport (3D FP)

Ray and beam tracing

Full wave analysis (multi mode)

Full wave analysis (FEM)

Wave dispersion

NBI analysis (birth, orbit, deposit)

Linear microinstability

Helical current evolution

Helical radial electric field

Neoclassical coefficient database

3D equilibrium and MHD stability

Drift-Kinetic equation solver

3D equilibrium
## Various Levels of Transport Modelling

### Fluid model

- **1D Diffusive transport equation:** $n(\rho, t), u_\phi(\rho, t), T(\rho, t)$
- **1D Dynamic transport equation:** $n(\rho, t), u(\rho, t), T(\rho, t)$
- **2D Dynamic transport equation:** $n(\rho, \chi, t), u(\rho, \chi, t), T(\rho, \chi, t)$
- **3D Gyrofluid equation:** $n(\rho, \chi, \zeta, t), u(\rho, \chi, \zeta, t), T(\rho, \chi, \zeta, t)$

### Kinetic model

- **Bounce-averaged drift-kinetic equation:** $f(p, \theta_p, \rho, t)$
- **Axisymmetric gyrokinetic equation:** $f(p, \theta_p, \rho, \chi, t)$
- **Gyrokinetic equation:** $f(p, \theta_p, \rho, \chi, \zeta, t)$
- **Full kinetic equation:** $f(p, \theta_p, \phi_g, \rho, \chi, \zeta, t)$
Transport Modelling in the TASK code

- **Diffusive transport equation: TASK/TR**
  - Diffusion equation for plasma density
  - Flux-Gradient relation
  - Conventional transport analysis

- **Dynamical transport equation: TASK/TX:**
  - Two-fluid equation and Maxwell’s equation
  - Flux-averaged fluid equation
  - Plasma rotation and transient phenomena

- **Kinetic transport equation: TASK/FP:**
  - Drift-kinetic equation for momentum distribution function
  - Bounce-averaged Fokker-Plank equation
  - Time evolution of momentum distribution
Diffusive Transport Equation: TASK/TR

- **Transport Equation Based on Gradient-Flux Relation:**

  \[ \Gamma = \vec{M} \cdot \frac{\partial F}{\partial \rho} \]

  where \( V \): Volume, \( \rho \): Normalized radius, \( V' = \frac{dV}{d\rho} \)

  - **Particle transport**

    \[ \frac{1}{V'} \frac{\partial}{\partial t} \left( n_s V' \right) = -\frac{\partial}{\partial \rho} \left( V' \langle |\nabla \rho| \rangle n_s V_s - V' \langle |\nabla \rho|^2 \rangle D_s \frac{\partial n_s}{\partial \rho} \right) + S_s \]

  - **Toroidal momentum transport**

    \[ \frac{1}{V'} \frac{\partial}{\partial t} \left( n_s u_{\phi s} V' \right) = -\frac{\partial}{\partial \rho} \left( V' \langle |\nabla \rho| \rangle n_s u_{\phi s} V_{M s} - V' \langle |\nabla \rho|^2 \rangle n_s \mu_s \frac{\partial u_{\phi s}}{\partial \rho} \right) + M_s \]

  - **Heat transport**

    \[ \frac{1}{V'^{5/3}} \frac{\partial}{\partial t} \left( \frac{3}{2} n_s T_s V'^{5/3} \right) = -\frac{1}{V'} \frac{\partial}{\partial \rho} \left( V' \langle |\nabla \rho| \rangle \frac{3}{2} n_s T_s V_{E s} - V' \langle |\nabla \rho|^2 \rangle n_s \chi_s \frac{\partial T_s}{\partial \rho} \right) + P_s \]

  - **Current diffusion**

    \[ \frac{\partial B_\theta}{\partial t} = \frac{\partial}{\partial \rho} \left[ \frac{\eta}{FR_0 \langle R^{-2} \rangle \mu_0} \frac{R_0}{V'} \frac{F^2}{\partial \rho} \left( \frac{V' B_\theta}{F} \langle |\nabla \rho|^2 \rangle \right) - \frac{\eta}{FR_0 \langle R^{-2} \rangle} \langle J \cdot B \rangle_{\text{ext}} \right] \]
Transport processes

* Neoclassical transport
  ‣ Collisional transport in a nonuniform magnetic field
  ‣ Radial diffusion, enhanced resistivity, bootstrap current, Ware pinch

* Turbulent transport
  ‣ Various transport models
  ‣ GLF23, CDBM, Bohm/gyro Bohm, TGLF, ...

* Atomic transport
  ‣ charge exchange, ionization, recombination

* Radiation transport
  ‣ Line radiation, Bremsstrahlung, Synchrotron radiation

* Parallel transport
  ‣ along open magnetic field lines in SOL plasmas

* Sources
  ‣ Particle: gas puff, NBI, pellet
  ‣ Momentum: NBI, waves
  ‣ Heat: NBI, waves, fusion reaction
Heat Transport Simulation of ITER Scenarios

High Performance Scenario

CDBM05

$\beta_N = 1.88$

$\tau_E = 3.0 \text{ s}$

Steady State Scenario

CDBM05

$\beta_N = 1.8$

$\tau_E = 3.1 \text{ s}$
Dynamical Transport Equations (TASK/TX)

- M. Honda and A. Fukuyama, JCP 227 (2008) 2808
- A set of flux-surface averaged equations
- Two fluid equations for electrons and ions
  - Continuity equations
  - Equations of motion (radial, poloidal and toroidal)
  - Heat transport equations
- Maxwell’s equations
- Slowing-down equations for beam ion component
- Diffusion equations for three-group neutrals

Self-consistent description of plasma rotation and electric field

- Equation of motion rather than transport matrix

Quasi-neutrality is not assumed.
Dynamical Transport Equation in TASK/TX (1)

- **Continuity equations:**

  \[
  \frac{\partial n_s}{\partial t} = -\frac{1}{r} \frac{\partial}{\partial r} (r n_s u_{sr}) + \frac{1}{r} \frac{\partial}{\partial r} \left( r D_m v_{Ts} \frac{\partial n_s}{\partial r} \right) + S_s
  \]

- **Equations of motion:**

  \[
  \frac{\partial}{\partial t} (m_s n_s u_{sr}) = -\frac{1}{r} \frac{\partial}{\partial r} (r m_s n_s u_{sr}^2) + \frac{1}{r} r m_s n_s u_{s\theta}^2 - \frac{\partial}{\partial r} (n_s T_s) + e_s n_s (E_r + u_{s\theta} B_\phi - u_{s\phi} B_\theta)
  \]

  \[
  \frac{\partial}{\partial t} (m_s n_s u_{s\theta}) = -\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 m_s n_s u_{sr} u_{s\theta}) + \frac{1}{r^2} \frac{\partial}{\partial r} \left[ r^3 m_s n_s u_s \frac{\partial}{\partial r} \left( \frac{u_{s\theta}}{r} \right) \right] + e_s n_s (E_\theta - u_{sr} B_\phi)
  \]

  \[
  + \frac{1}{r} \frac{\partial}{\partial r} \left[ r D_m v_{Ts} \frac{\partial}{\partial r} (m_s n_s u_{s\theta}) \right] + F_{s\theta}^{NC} + F_{s\theta}^{HNC} + F_{s\theta}^C + F_{s\theta}^W + F_{s\theta}^L + F_{s\theta}^N + F_{s\theta}^{CX}
  \]

  \[
  \frac{\partial}{\partial t} (m_s n_s u_{s\phi}) = -\frac{1}{r} \frac{\partial}{\partial r} (r m_s n_s u_{sr} u_{s\phi}) + \frac{1}{r} \frac{\partial}{\partial r} \left( r m_s n_s u_s \frac{\partial u_{s\phi}}{\partial r} \right) + e_s n_s (E_\phi + u_{sr} B_\theta)
  \]

  \[
  + \frac{1}{r} \frac{\partial}{\partial r} \left[ r D_m v_{Ts} \frac{\partial}{\partial r} (m_s n_s u_{s\phi}) \right] + F_{s\phi}^{HNC} + F_{s\phi}^C + F_{s\phi}^W + F_{s\phi}^L + F_{s\phi}^N + F_{s\phi}^{CX}
  \]
Dynamical Transport Equation in TASK/TX (2)

- **Heat transport equations:**

\[
\frac{\partial}{\partial t} \left( \frac{3}{2} n_s T_s \right) = -\frac{1}{r} \frac{\partial}{\partial r} \left( \frac{5}{2} r u_s n_s T_s - \frac{3}{2} r n_s \chi_s \frac{\partial T_s}{\partial r} \right) + e_s n_s (E_\theta u_{s\theta} + E_\phi u_{s\phi})
\]

\[
+ \frac{1}{r} \frac{\partial}{\partial r} \left[ r D_m v_T (n_s t_s) \right] + P_s^C + P_s^L + P_s^R + P_s^{RF}
\]

- **Maxwell’s equation**

\[
\frac{1}{R} \frac{\partial}{\partial R} (R E_r) = \frac{1}{\varepsilon_0} \sum_s e_s n_s
\]

\[
\frac{1}{c^2} \frac{\partial E_\theta}{\partial t} = -\frac{\partial B_\phi}{\partial r} - \mu_0 \sum_s e_s n_s u_{s\theta}
\]

\[
\frac{1}{c^2} \frac{\partial E_\phi}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} (r B_\theta) - \mu_0 \sum_s e_s n_s u_{s\phi}
\]

\[
\frac{\partial B_\theta}{\partial t} = \frac{\partial E_\phi}{\partial r}, \quad \frac{\partial B_\phi}{\partial t} = -\frac{1}{r} \frac{\partial}{\partial r} (r E_\theta)
\]
Typical Ohmic Plasma Profiles at $t = 50$ ms

**JFT-2M like plasma** composed of electron and hydrogen

$R = 1.3$ m, $a = 0.35$ m, $b = 0.4$ m, $B_{\phi b} = 1.3$ T, $I_p = 0.2$ MA, $S_{\text{puff}} = 5.0 \times 10^{18}$ m$^{-2}$s$^{-1}$

$\gamma = 0.8$, $Z_{\text{eff}} = 2.0$, Fixed turbulent coefficient profile
Density Profile Modification Due to NBI Injection

Modification of \( n \) and \( E_r \) profile depends on the direction of NBI.

Co/Counter with \( I_p \): Density flattening/peaking
Toroidal Rotation Due to Ion Orbit Loss

- Ion orbit loss near the edge region drives toroidal rotation

Ref. M. Honda et al., NF (2008) 085003
Kinetic Integrated Modelling: Motivation

- Better understanding of burning plasmas
  - Behavior of energetic particles
    - generation, transport excitation

- Analysis of momentum distribution function
  - Consistent analysis of heating and current drive
    - both bulk and energetic components
    - all heating schemes
  - Influence of energetic particles on heating processes
    - propagation and absorption of waves
    - fusion reaction rate
  - Modification of momentum distribution due to radial transport

- Modelling based on momentum distribution function is required.
Fokker-Planck Analysis in TASK/FP

- **Multi-species momentum distribution functions:**
  \[ f_s(p_\parallel, p_\perp, \rho, t) \]

- **Fokker-Planck equation**
  \[ \frac{\partial f_s}{\partial t} = E(f_s) + C(f_s) + Q(f_s) + D(f_s) + S_s \]
  - **\( E(f) \):** Acceleration due to DC electric field
  - **\( C(f) \):** Relativistic Non-Maxwellian Coulomb collision
  - **\( Q(f) \):** Quasi-linear diffusion due to wave-particle resonance
    - Full wave analysis (TASK/WM)
    - Ray/beam tracing (TASK/WR)
    - Fixed wave field profile; Fixed diffusion coefficient profile
  - **\( D(f) \):** Spatial diffusion
  - **\( S \):** Particle Source and Sink (NBI, Fusion reaction)
**Kinetic Transport Modelling: TASK/FP**

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<thead>
<tr>
<th>Feature</th>
<th>Description</th>
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<tr>
<td>Multi species</td>
<td>conservation between species</td>
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<tr>
<td>Three dimensional</td>
<td>2D in momentum, 1D in radial</td>
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<td>Bounce averaged</td>
<td>trapped particle effect</td>
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<td>Nonlinear collision</td>
<td>momentum and energy conservation</td>
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<td>Relativistic</td>
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<td>Fusion reaction</td>
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<td>Parallel processing</td>
<td>using parallel matrix solver PETSc library</td>
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<td>Finite orbit size</td>
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<td>Induced EM fields</td>
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Multi-Species Fokker-Planck Analysis

Momentum distribution functions:

Electron : EC+LH

D : NBI

T : ICRF

He : DT reaction
Analysis of Multi-Scheme Heating in ITER Plasma

• **2D MHD equilibrium**
  - \( R = 6.2 \) m, \( a = 2.0 \) m, \( \kappa = 1.7 \), \( \delta = 0.33 \), \( B_0 = 5.3 \) T, \( I_p = 3 \) MA

• **Multi species:**
  - Electron, D, T, He

• **Multi scheme heating:**
  - ICH, NBI, NF (DT, DD, TT)

• **Initial density:**
  - \( n_e(0) = 10^{20} \text{ m}^{-3} \), \( n_D(0) = 5 \times 10^{19} \text{ m}^{-3} \), \( n_T(0) = 5 \times 10^{19} \text{ m}^{-3} \)

• **Initial temperature:**
  - \( T_e(0) = T_D(0) = T_T(0) = 20 \text{ keV} \)

• **Radial diffusion coefficient:** simplest model
  - \( D_{rr} = 0.1(1 + 9\rho^2) \text{ m/s} \)
Momentum Distribution Functions (t = 1 s)
Power Transfer between Species

- **Collisional power transfer**

  - **Electron**
    - ICH $\downarrow 8.4$
    - $1.1\pm0.2$
    - $40.1\pm0.5$
    - $3.1\pm0.1$
  - **D**
    - NBI $\downarrow 31.6$
    - $11.0\pm1.8$
    - $7.2\pm0.1$
  - **T**
    - ICH, NF(DD) $\uparrow 18.17 \pm 0.5$
    - $4.8\pm0.1$
  - **He**
    - NF(DT) $\uparrow 61.3$

- **Requires more momentum meshes for better accuracy**
  - At present, typically $100 \times 100 \times 50$
Simulation with Radial Transport

**Absorbed power vs \( \rho \)**

- \( P_{\text{abs}} \) [MW/m²]
- \( \rho \)
- Parameters: \( T \), \( \alpha \), \( e \), \( D \)

**Kinetic energy density vs \( \rho \)**

- \( T \) [keV]
- \( \rho \)
- Parameters: \( T \), \( e \), \( D \)

**Collisional power transfer vs \( t \)**

- \( P_c \) [MW]
- \( t \) [ms]
- Parameters: \( T \), \( e \), \( D \), \( \alpha \)

**Collisional power transfer vs \( \rho \)**

- \( P_c \) [MW/m²]
- \( \rho \)
- Parameters: \( T \), \( e \), \( D \), \( \alpha \)
Dependence on Radial Diffusion model

Radial profile of average kinetic energy:

- **$p$-indep. diffusion**
  
  $$D_{rr} \propto p^0$$

- **$p$-dep. diffusion**
  
  $$D_{rr} \propto p^{-1}$$

- **no diffusion**
  
  $$D_{rr} = 0$$

\[ p' = \sqrt{p^2 + p_{th0}^2} \]

<table>
<thead>
<tr>
<th></th>
<th>$p_d$ [keV]</th>
<th>$p_t$ [keV]</th>
<th>$E_{K\alpha}$ [keV]</th>
<th>$P_{NB}$ [MW]</th>
<th>$P_{ICT}$ [MW]</th>
<th>$P_\alpha$ [MW]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{Kd}$ [keV]</td>
<td>9.57</td>
<td>10.82</td>
<td>471.70</td>
<td>31.68</td>
<td>8.95</td>
<td>23.36</td>
</tr>
<tr>
<td>$E_{KT}$ [keV]</td>
<td>7.18</td>
<td>8.15</td>
<td>558.28</td>
<td>31.68</td>
<td>10.87</td>
<td>32.70</td>
</tr>
<tr>
<td>$E_{K\alpha}$ [keV]</td>
<td>11.72</td>
<td>9.44</td>
<td>622.75</td>
<td>31.69</td>
<td>15.28</td>
<td>36.88</td>
</tr>
</tbody>
</table>

- Radial diffusion proportional to $E^{-1/2}$ reduces the alpha heating about 10%.
Full Wave Analysis

• Boundary-value problem of Maxwell’s equation with fixed $\omega$
  – $E$: wave electric field
  – $\varepsilon$: dielectric tensor

$$\nabla \times \nabla \times E = \frac{\omega^2}{c^2} \varepsilon \cdot E + i \omega \mu_0 j_{ext}$$

• Merit of full wave analysis
  – Wave length longer than the scale length of medium
  – Propagation over an evanescent layer
  – Coupling to antenna
  – Formation of standing wave

• Method of full wave analysis
  – Fourier analysis: algebraic equation
  – Discrete differential equation: finite difference/element method
  – Mixture of above two methods
Full wave analysis: TASK/WM

- Maxwell’s equation solver as a boundary-value problem
  \[ \nabla \times \nabla \times E = \frac{\omega^2}{c^2} \vec{\varepsilon} \cdot E + i \omega \mu_0 \vec{j}_{\text{ext}} \]

- Kinetic dielectric tensor: \( \vec{\varepsilon} \) for arbitrary \( f(v) \)

- Numerical scheme: Fourier expansion in \( \theta \) and \( \phi \)

- Antenna excitation and eigenmode analysis: Complex \( \omega \)

- ICRF minority heating

\[\text{Im}(E_{\text{th}})\quad P_{\text{He3}}\quad P_{\text{abs}}(r)\]
ICRF Waves in a Helical Plasma

**LHD** ($B_0 = 3$ T, $R_0 = 3.8$ m)

\[ f = 42 \text{ MHz}, \quad n_{\phi 0} = 20, \quad n_{e0} = 3 \times 10^{19} \text{ m}^{-3}, \quad n_H/(n_{\text{He}} + n_H) = 0.235, \]

\[ N_{r_{\max}} = 100, \quad N_{\theta_{\max}} = 16 \ (m = -7 \ldots 7), \quad N_{\phi_{\max}} = 4 \ (n = 10, 20, 30) \]

**Wave electric field** (imaginary part of poloidal component)

**Power deposition profile** (minority ion)
TAE Analysis with TASK/WM

- Configuration
  - $q(\rho) = q_0 + (q_a - q_0)\rho^2$, $q_0 = 1$, $q_a = 2$
  - Flat Density Profile

Contour of $|E|^2$ in Complex Frequency Space

Alfvén Frequency

Eigen function

$f_r = 81.95 \text{ kHz}$
$f_i = -20.32 \text{ Hz}$
RSAE Excitation by Energetic Particles

- **Without EP**
  - $q_{\min} = 2.6$, $f_r = 37.5$ kHz
  - $m = -3$

- **With EP**
  - $3 \times 10^{16}$ m$^{-3}$
  - 360 keV
  - 0.5 m
  
  - $m = 3$
  - $f_r = 38.0$ kHz
  - $f_i = 160.2$ Hz

- **With EP**
  - $1 \times 10^{17}$ m$^{-3}$
  - 360 keV
  - 0.5 m
  
  - $m = 3$
  - $f_r = 37.2$ kHz
  - $f_i = 1858.6$ Hz

- **With EP**
  - $n_e = 0$ m$^{-3}$
  - $m = -2$
  - $m = -3$

- **With EP**
  - $n_e = 3 \times 10^{16}$ m$^{-3}$
  - $f_r = 55.3$ kHz
  - $f_i = 75.7$ Hz

- **With EP**
  - $n_e = 1 \times 10^{17}$ m$^{-3}$
  - $f_r = 55.3$ kHz
  - $f_i = 271.6$ Hz

- **With EP**
  - $n_e = 0$ m$^{-3}$
  - $m = 2$
  - $m = 3$
Progress in Full Wave Analysis

- **Variety of numerical schemes**

<table>
<thead>
<tr>
<th>module</th>
<th>system</th>
<th>scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>WM</td>
<td>torus</td>
<td>toroidal &amp; poloidal: FFT, radial: FDM</td>
</tr>
<tr>
<td>WMF</td>
<td>torus</td>
<td>toroidal &amp; poloidal: FFT, radial: FEM</td>
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<tr>
<td>WF2D</td>
<td>torus</td>
<td>toroidal: FFT, poloidal and radial: FEM</td>
</tr>
<tr>
<td>WF3D</td>
<td>Cartesian</td>
<td>$x, y, z$: FEM</td>
</tr>
</tbody>
</table>

- Merit of FEM: Flexibility of mesh, sparse matrix, localized analysis

- **Extension of dielectric tensor**

  - Uniform, kinetic, Maxwellian, Fourier expansion
  - Nonuniform, gyro kinetic, Maxwellian, Fourier expansion
  - Nonuniform, kinetic, Maxwellian, Integral form
  - Uniform, kinetic, arbitrary $f(v)$, Fourier expansion
  - Nonuniform, gyro kinetic, arbitrary $f(v)$, Fourier expansion

- **Coupling with Fokker-Planck analysis of $f(v)$**
• Wave electric field with complex frequency:  \( \tilde{E}(r, t) = E(r) e^{-i \omega t} \)

• Maxwell’s equation:  \( \nabla \times \nabla \times E - \frac{\omega^2}{c^2} \varepsilon \cdot E = i \omega \mu_0 j_{\text{ext}} \)
  – \( \varepsilon \): Dielectric tensor
    ◦ Collisional cold plasma model

• Numerical method: FEM
  – 3D version
    ◦ Tetrahedron element
    ◦ Electric field along a edge of a tetrahedron
  – 2D version: axisymmetric cylindrical
    ◦ Triangular element
    ◦ Scalar (toroidal) and vector (poloidal) hybrid basis function
EC waves in a small-size ST

- $R=0.22 \text{ m}$, $a=0.16 \text{ m}$, $B_0=0.072 \text{ T}$
- $f=5 \text{ GHz}$, $n_\varphi=8$, $v/\omega=0.001$

\[
\begin{array}{cccccc}
 n_0 & 10^{17} \text{ m}^{-3} & 2 \times 10^{17} \text{ m}^{-3} & 3 \times 10^{17} \text{ m}^{-3} & 4 \times 10^{17} \text{ m}^{-3} & 5 \times 10^{17} \text{ m}^{-3} \\
 \text{Im } E_\theta \quad \text{(X-mode)} & & & & & \\
 \text{Im } E_\phi \quad \text{(O-mode)} & & & & & \\
 P_{\text{abs}} & & & & & \\
 \end{array}
\]

Legend:
- ECR layer
- UHR layer
- XR-mode cutoff layer
- O-mode cutoff layer
- XL-mode cutoff layer
Integral Formulation of Wave-Particle Interaction

- General form of dielectric tensor
  \[ \nabla \times \nabla \times E(r, \omega) - \frac{\omega^2}{c^2} \int_V \mathrm{d}r' \left[ \mathcal{E}(r, r'; \omega) \cdot E(r', \omega) - i \frac{\omega}{\mu_0} J_{\text{ext}}(r, \omega) \right] = 0 \]

- Particle orbit:
  \[ r = r' + \Delta r(v, r, t - t') \]
  \[ v = v' + \Delta v(v, r, t - t') \]

- Perturbed distribution from Vlasov equation:
  \[ f(r, v, t) = -\frac{q}{m} \int_{-\infty}^{t} \mathrm{d}t' \left[ E(r') + v' \times B(r') \right] \cdot \frac{\partial f_0(r', v')}{\partial v'} e^{-i \omega t'} \]

- Induced current:
  \[ j(r) = \int \mathrm{d}v q v f(r, v, t) e^{i \omega t} = \int \mathrm{d}r' \mathcal{\Sigma}(r - r', t - t') \cdot E(r') \]

- The integral form of the conductivity tensor is defined by
  \[ \mathcal{\Sigma}(r, r', t-t') = -\frac{q}{m} \int_{-\infty}^{t} \mathrm{d}t' \left[ \frac{\partial f_0(r', v')}{\partial v'} \right] \left[ v + \frac{1}{i \omega} v' \times \nabla \times \right] \left|_{r' = r - \Delta r(v, r, t - t')} v' = v - \Delta v(v, r, i - t')} \right] \]
Variable Transformation

- **Transformation of Integral Variables**
  - Transformation from the velocity space variables \((v_\perp, \theta_g)\) to the particle position \(s'\) and the guiding center position \(s_0\).
  - Jacobian: \(J = \frac{\partial (v_\perp, \theta_g)}{\partial (s', s_0)} = -\frac{\omega_c^2}{v_\perp \sin \omega_c \tau}\).
  - Express \(v_\perp\) and \(\theta_g\) by \(s'\) and \(s_0\) using \(\tau = t - t'\), e.g.,
    \[
    v_\perp \sin(\omega_c \tau + \theta_g) = \frac{\omega_c}{v_\perp} \left( \frac{s - s'}{2} \tan \frac{1}{2} \omega_c \tau \right) + \frac{\omega_c}{v_\perp} \left( \frac{s + s'}{2} - s_0 \right) \tan \frac{1}{2} \omega_c \tau
    \]

- **Integration over \(\tau\)**: Fourier expansion with cyclotron motion
- **Integration over \(v_\parallel\)**: Plasma dispersion function
- **Conductivity tensor**: \((\ell : \text{cyclotron harmonics number})\)
  \[
  \tilde{\sigma}(s, s', x_0, \zeta_0) = -in_0 \frac{q^2}{m} \sum_{\ell} \int ds_0 \tilde{H}_\ell(s - s_0, s' - s_0; s_0, x_0, \zeta_0)
  \]
Kernel Functions

- Kernel Function $H_\ell$ and its integral in FEM includes:

$$F_n^{(i)}(X, Y) = \frac{1}{2\pi^2} \int_0^\pi d\theta \exp \left[ - \frac{X^2}{1 + \cos \theta} - \frac{Y^2}{1 - \cos \theta} \right] f_n^{(i)}(\theta)$$

$$F_\ell^{(ij)}(X, Y, Y') \equiv \int_0^Y dY' \int_0^{X+Y'} dX' X'^i Y'^j F_n^{(i)}(X', Y')$$

$$f_n^{(i)}(\theta) = \begin{cases} 
\frac{\cos n\theta}{\sin \theta} & (i = 1) \\
\sin n\theta & (i = 2) \\
\frac{\sin n\theta}{\sin^2 \theta} & (i = 3) \\
\frac{\cos \theta \sin n\theta}{\sin^2 \theta} & (i = 4)
\end{cases}$$
One-Dimensional Analysis

O-X-B excitation

a small-size spherical tokamak

major radius \( R_0 = 0.22 \) m
minor radius \( a = 0.15 \) m
central magnetic field \( B_0 = 0.08 \) m

toroidal mode number \( n_\phi = 24 \)
central electron density \( 3 \times 10^{17} \) m\(^{-3}\)

\( T_e(0) = 2 \) kev

\( T_e(0) = 5 \) kev
Issues in Kinetic Integrated Modeling

* Modeling of transport process
  ‣ Turbulent transport coefficients with velocity dependence
  ‣ Finite orbit size effects (Neoclassical transport)
  ‣ Coupling with toroidal electric field (Faraday’s law)
  ‣ Keeping charge neutrality (Gauss’s Law)

* Kinetic full wave analysis
  ‣ Integral form of dielectric tensor including finite gyro radius effects
  ‣ Gyro kinetic dielectric tensor for coupling with drift waves

* Coupling with other components
  ‣ Equilibrium including kinetic effects
    • Anisotropic pressure, and flow
  ‣ Modeling of diagnostics
    • Validation by direct comparison
Summary

* Integrated modelling of toroidal plasmas is required for understanding the physics of experimental observations and predicting the performance of future devices.

* For large scale integrated simulation, development and spread of a standard data model is essential. Several efforts to develop infrastructures for integrated modelling are under way.

* We have been developing the integrated modelling suites TASK which includes several levels of transport modelling and full wave analysis of toroidal plasmas.